Measurement and Data:
Volume
Grade 5
Formative Assessment Lesson

Designed and revised by the Kentucky Department of Education
Field-tested by Kentucky Mathematics Leadership Network Teachers

If you encounter errors or other issues with this file, please contact the KDE math team at:
kdemath@education.ky.gov

Revised 2017

This formative assessment lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should be used to inform the instruction that will take place for the remainder of the unit.

Mathematical goals

This lesson is intended to help you assess how well students are able to model three dimensional figures and find their volume. In particular, this unit aims to identify and help students who have difficulties with:

- Recognizing volume as an attribute of three-dimensional space.
- Measuring volume by finding the total number of same-size units of volume required to fill the space without gaps or overlaps.
- Measuring necessary attributes of shapes, in particular the base area, in order to determine volumes to solve real world and mathematical problems.

Kentucky Academic Standards

This lesson involves *mathematical content* in the standards from across the grade, with emphasis on:

Measurement and Data 5.MD

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

This lesson involves a range of *Standards for Mathematical Practice* with emphasis on:

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
7. Look for and make use of structure.

Introduction

This lesson unit is structured in the following way:

• Before the lesson, students work individually on an assessment task that is designed to reveal their current understanding and difficulties. You then review their work and formulate questions for students to answer to help them improve their solutions.

• After a whole class introduction, students work in pairs to match the word problem and models of the 3-dimensional figures. Throughout their work, students justify and explain their decisions to their peers.

• Toward the end of the lesson there is a whole class discussion.

• Finally, students return to their original assessment task, and try to improve their own responses.

Materials Required

- Each individual student will need two copies of the worksheet *How Many Cubes?*
- Each individual student will need a copy of the graphic organizer to be used in the whole class introduction.
- Each partner group of students will need a packet of Card Set A and B.
- Each partner group of students will also need at least 30 cubes or blocks so that they can model the word problems if needed. You may provide tiles as well. Then students can practice/experience selecting the correct manipulative needed for this task. Tiles will not be effective for this "cube" volume task.

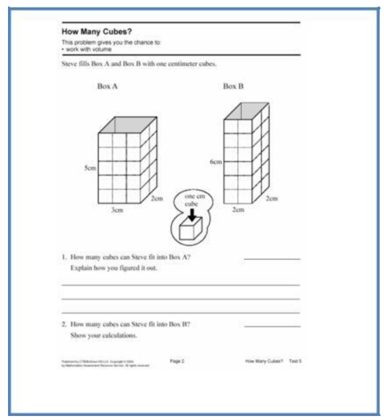
Time Needed

Approximately fifteen minutes for the assessment task given a few days prior to the lesson, a one-hour lesson, and 15 minutes for the students to review their work for changes. All timings are approximate. Exact timings will depend on the needs of the class.

Before the Lesson

Assessment task: How Many Cubes?

- Have the students do this task in class a day or more before the Formative Assessment Lesson.
- This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.



How Many Cubes?

This problem gives you the chance to:
• work with volume

Steve fills Box A and Box B with one centimeter cubes.

1. How many cubes can Steve fit into Box A? ______
Explain how you figured it out.

2. How many cubes can Steve fit into Box B? ______
Show your calculations.

Page 2 How Many Cubes? Test 5

<u>Framing the pre-assessment: (10-15 minutes)</u>

Give each student a copy of *How Many Cubes?* Introduce the task briefly and help the class to understand the problem and its context.

> ***Spend ten-fifteen minutes on your own, answering this questions.***
>
> ***Don't worry if you can't figure it out. There will be a lesson on this material [tomorrow] that will help you improve your work. Your goal is to be able to answer these questions with confidence by the end of that lesson.***

It is important that students complete the task without assistance, as far as possible.

If students are struggling to get started, ask them questions that help them understand what is required, but do not do the task for them.

<u>Assessing Students' Responses:</u>

- Collect students' responses to the task. <u>We suggest you do not actually score student's work.</u> The research shows this will be counterproductive, as it will encourage students to compare their scores, and will distract attention from how they may improve their mathematics. Make some notes on what their work reveals about their current levels of understanding as they figure out the volume of the boxes. The purpose of this is to forewarn you of the issues that will arise during the lesson, so that you may prepare carefully.

- Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some questions in the *Common Misconceptions* chart may serve as examples. These questions have been developed from commonly identified student misconceptions.

- Make notes about what their work reveals about their current levels of understanding and their different problem solving approaches.
- Partner students with others who displayed similar errors/misconceptions on the pre-assessment task.
- Remember all students may not get through all sets of cards and that is okay.

We recommend you:

- Write one or two questions on each students' work, or
- Give each student a printed version of your list of questions and highlight the questions for each individual student or partner group, or
- Display a small list of questions on the board that will be of help to the majority of students and review over the list before the card sort, or
- The solution to all these difficulties is not to teach one particular way of solving a problem, but to help students to find a variety of ways that work in different situations and make sense to them.

Below is a list of common misconceptions and questions/prompts that may be written on individual student papers, or displayed in the room or on table areas, or with partners.
We suggest that you write your own lists of questions, based on your own students' work, using the ideas below.

Common issues: Suggested questions and prompts:

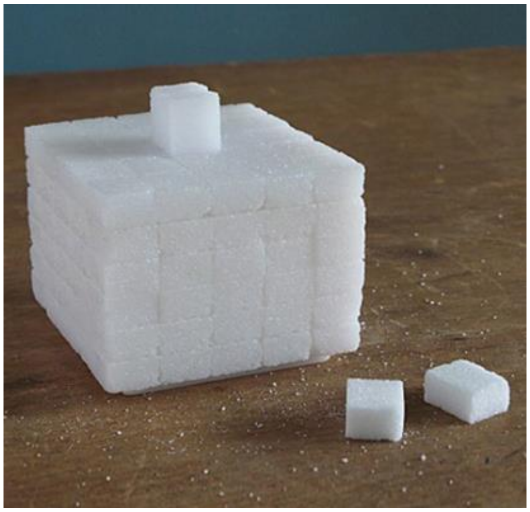
Common Issues	Suggested questions and prompts
Student who has trouble getting started.	• *What information do you know?* • *How can you use what you know to begin the problem?*
Student confuses area and volume because they do not understand what each describes.	• *How many cubes will fit into a prism this size?* • *Compare this problem/model to the area problems/models you have seen. How is this similar? How is this different?*
Student does not connect the 3 dimensional model cards to the word problem.	▪ *How can you use the cubes provided to build a model? Which card from set B matches your model?*
Student does not see how the base area can be used to find volume.	▪ *How many cubes will fit in the bottom layer? What does this represent?* ▪ *What if you know how many layers are in the model? Can you use this to help you find the volume?*

Suggested Lesson Outline

Whole Class Introduction (10 minutes)

The student misconception data from the pre-assessment will drive this whole class introduction. The purpose of this whole class introduction is to tap into student interest and to focus student thinking about the content, NOT to reteach the concept or to come to a resolution about this problem. It is important to let students share their thinking without leading them through the process to the conclusion.

Display the following image:



Say, "*Today we are going to do some more work with solving volume problems".* Display the image.

Ask students *"What do you notice? What do you wonder?"* Students share ideas with shoulder partner. Select a few students to share aloud and record their noticings.

Ask, *"How might we figure out how many sugar cubes are in the image?"* Students share ideas with shoulder partner and select a couple to share aloud. It is IMPORTANT TO NOT HAVE A RETEACH LESSON. AT THIS POINT, THIS IS JUST INQUIRY AS STUDENTS BEGIN THE COLLABORATIVE ACTIVITY.

Collaborative activity 1 – Matching Card Set A *Task Cards* and Card Set B *3-D Models (15 minutes)*

Organize the students into partners of two or three based on common misconceptions from the preassessment task. In trials, teachers found keeping small homogenous partners helped more students play an active role.

Introduce the lesson carefully:

> ***I want you to work as a team. Begin with a Task card from Card Set A.***
>
> ***Model this problem with the blocks first. (optional step, you may want to let students decide if they want to model first)***
>
> ***Then find a card from Card Set B* that matches the model you built.**
>
> ***Continue this with all task cards. Each time you do this; explain your thinking clearly and carefully to your partner. Justify your thinking.***
>
> ***If your partner disagrees with the model you chose, then challenge him/her. It is important that you both understand the math for all the models.***
>
> ***There is a lot of work to do today, and it doesn't matter if you don't all finish. The important thing is to learn something new, so take your time.***

You have two tasks during partner work, 1) to note different student approaches to the task, and 2) to support student problem solving. You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes.

Note different student approaches to the task

Listen and watch students carefully. In particular, listen to see whether they are addressing the difficulties outlined in the *Common Issues* table. You can use this information to focus a whole-class discussion towards the end of the lesson.

Support student problem solving

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. If several students in the class are struggling with the same issue, you could write a relevant question on the board. You might also ask a student who has performed well on one part of the task to help a student struggling with that part of the task.

The following questions and prompts would be helpful:

What information have you been given?

What do you need to find out?

How can you model the 3-dimensional figure that was given in the problem?

If one student has modeled with a set of bar model cards, challenge their partner to provide an explanation.

Maria modeled the problem with these cards. Martin, why does Maria model it this way?

If you find students have difficulty articulating their decisions, then you may want to use the questions from the *Common Issues* table to support your questioning.

Collaborative Activity 2: Placing *Card Set C: Base Area Cards (15 minutes)*

As students finish matching the word problem and model cards hand out Card Set C: *Base Area*. (Do not collect *Card Set A and B*.) This set of cards provides students with an opportunity to focus on the base of the prism. An important part of this cluster of 5th grade standards is that students discover that the volume of a prism is the base area x height. This will lead to the understanding that the volume of other 3 dimensional shapes is base area x height.

Collaborative Activity 3 : Placing CARD SET D – *V = L x W x H formula cards (15 minutes)*

As students finish placing the Base Area cards hand out Card Set D: *Formula Cards*. These provide students with a different way of modeling the situation with a numerical equation. Do not collect any of the previous cards.

Whole-class discussion comparing different approaches (15 minutes)

Organize a whole-class discussion to allow students to explain their models. The intention is for you to focus on getting students to understand the representations of the task to build their conceptual understanding of volume rather than showing them the formula. Focus your discussion on parts of the small-group tasks students found difficult.

Improve individual solutions to the assessment task (10 minutes)

Return to the students their original assessment, *How Many Cubes?* as well as a second blank copy of the task.

Look at your original responses and think about what you have learned this lesson.

Using what you have learned, try to improve your work.

If you have not added questions to individual pieces of work then display your list of questions on the board.
Students should select from this list only the questions appropriate to their own work.
If you find you are running out of time, then you could set this task in the next lesson.

This Formative Assessment Lesson was created around tasks taken from *Inside Mathematics.*

How Many Cubes? Answer KEY

(Manipulatives: Wooden cubes, plastic snap cubes to model this problem.)

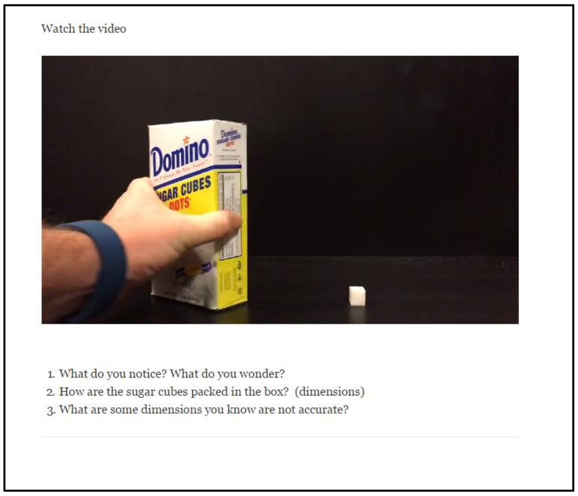
1. Box A can hold 30 cubes. A sample explanation could be that the bottom layer would consist of 6 cubes and 5 layers of 6 cubes would be 30 cubes. (Other explanations should be accepted.)
2. Box B can hold 24 cubes. A sample calculation – The base can be found by 2 x 2 is 4 cubes. Since there are 6 layers, 4 x 6 is 24 cubes.
3. Box A can hold more cubes.
4. This box can hold 36 cubes.

5. Accept any combination of numbers whose product is 36, i.e., 4, 3, 3.

Extension/follow Up Lesson

Each student will need a copy of the 3 Act Graphic Organizer from page 17.

The picture below is hyperlinked to the 3 Act Task "Sugar Cubes" from Graham Fletcher.



General Outline of 3 Act Task

Act 1: Introduce the central conflict of the story/task clearly and visually, using as few words as possible.

Act 2: The student overcomes obstacles, looks for resources, and develops new tools.

Act 3: Resolve the conflict.

Act 1:

Tell students they are going to watch a short video clip about sugar cubes packed in a box. Ask students to keep the following questions in mind as they watch the clip.

- ***What do you notice? What do you wonder?***
- ***How are the sugar cubes packed in the box? (dimensions)***
- ***What are some dimensions you know are not accurate?***

Show the clip to students.
Students complete boxes 1-6 on the graphic organizer independently.

Share noticings with a partner.
Ask and record a few answers on board/chart paper:

What are some things you noticed about the video?
What are some things you wondered about the video?
What question are we trying to answer? How are the sugar cubes packed in the box?
What are some possible dimensions you recorded in box #4?
What estimates might be too low? What estimates might be too high?

Act 2: Show the image below from Act 2 and the video clip. This image tells how many cubes are in the box total.

Ask: ***How does this information help us to determine how the cubes are packed in the box?***

Encourage students to revisit their responses in the graphic organizer with a partner for 1 minute.

Have a few students share out new thinking about the problem.

Show the 14 second video in Act 2.

Say***: So this is the base of the shape, we see 18 cubes and we know there are 198 cubes in the box, describe how will the remaining cubes fit in the box. Working with your partner, complete box 7-8 on the organizer. Note:*** You want students to come to their own conclusion that the base is a layer in the box and they are trying to find out how many equivalent layers are in the box.

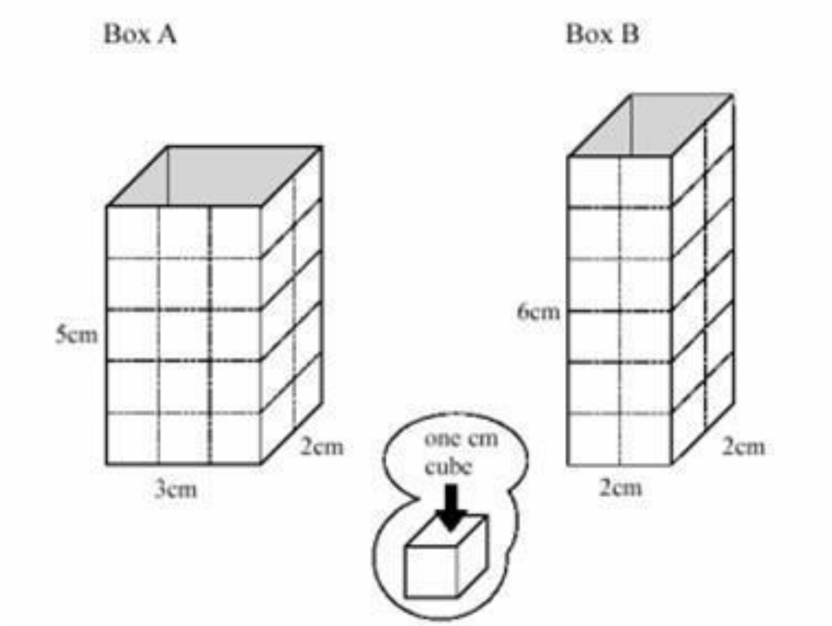
At this point it is not important that students are correct, what is important is for several students to share how they are thinking about the problem (strategies) with the whole group. Select students that demonstrate different strategies, even if they are not totally correct. It's the thinking that is important. Use a document camera, if possible, to display student work visually. You are not to reteach the content, just ask questions to clarify thinking or pose questions that cause students to think deeper about the content. The collaborative activity below is designed to clarify continued misconceptions.

Act 3: Show the image of how the sugar is packed and have students quickly discuss what they see with their partner.

Assessment Task *Name*______________________ *Date*_____________

How Many Cubes?

Steve fills Box A and Box B with one centimeter cubes.



1. How many cubes can Steve fit into Box A? ______________________

   Explain how you figured it out. __________________________________

   ____

   ____

2. How many cubes can Steve fit into Box B? ______________________

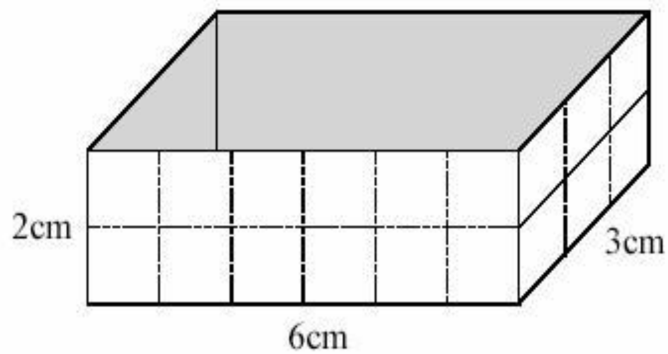
   Show your calculations.

3. Which of the two boxes can hold more cubes? ________________

4. Here is another box.

How many centimeter cubes can this box hold?

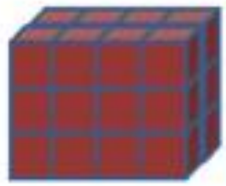
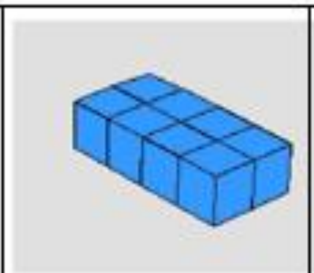
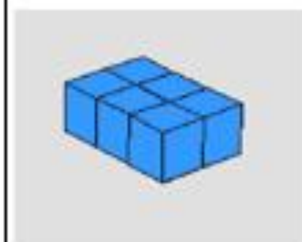
____



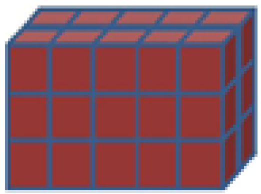
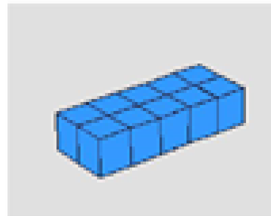
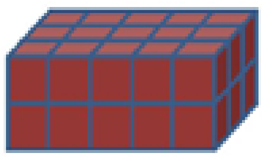
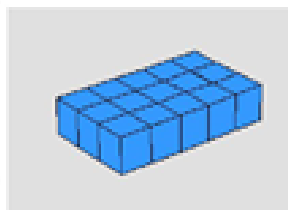
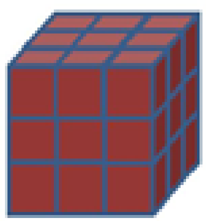
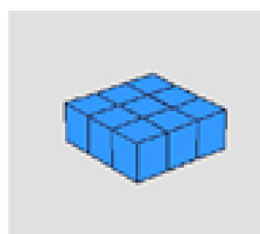
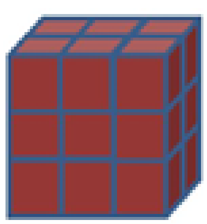
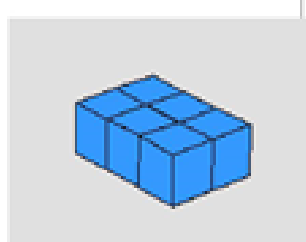
Find the measurements of a different box that holds the same number of cubes as this box.

__________ cm long __________ cm wide __________ cm high

Sample Solutions for Card Sort

A rectangular aquarium will hold 24 cubic feet of water when filled to the top. If it is 4 ft. long and 3 ft. tall, how wide is the tank?			V = 4 x 2 x ___
What is the volume of a rectangular prism with a height of 2 ft. and a base area of 6 ft^2?			V = ___ x 2 x 2
Cube-shaped boxes of candy are shipped in larger boxes. The larger boxes are six feet long, one foot wide, and two feet high. How many one cubic foot boxes of candy will the large box hold?			12 = 6 x ___ x ___
A rectangular juice box contains 24 milliliters of apple juice. The box is 2 cm high and 3 cm wide. What is the length of the juice box? (1 milliliter = 1 cubic centimeter)			24 = ___ x 3 x ___

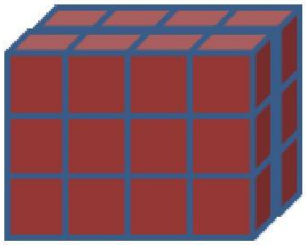
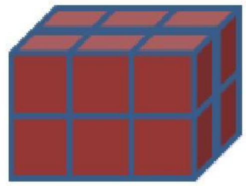
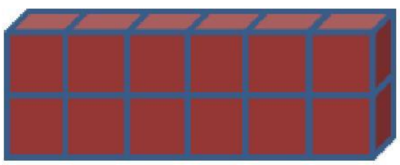
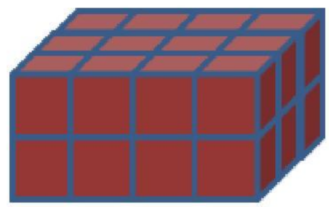
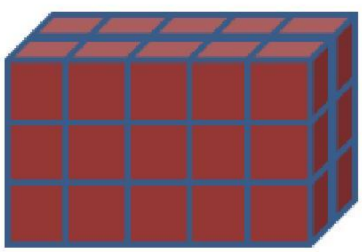
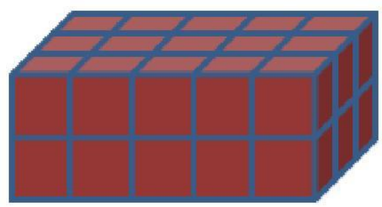
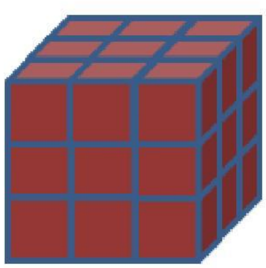
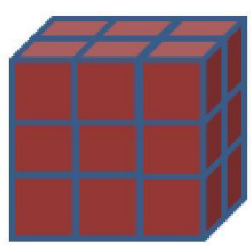
Sample solutions continued…

A wading pool will be 2 m long and 5 m wide. If it will hold 30 cubic meters of water, how deep should this wading pool be?			V = 5 x __ x 3
A moving company is trying to store 1 cubic meter boxes in a storage room with a length of 5 m, width of 3 m and height of 2 m. How many boxes can fit in this space?			30 = _ x 3 x __
What is the volume of a cube with an edge that measures 3 cm?			V = 3 x 3 x __
A toy company is planning to market a set of wooden alphabet blocks. Each block is a cube with 1-inch edges. How many blocks will fit into a container that is 3 cm long, 2 cm wide, and 3 cm tall?			18 = _ x 2 x __

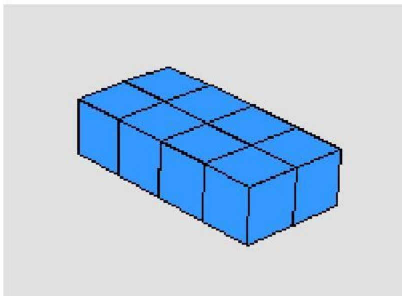
CARD SET A: TASK CARDS

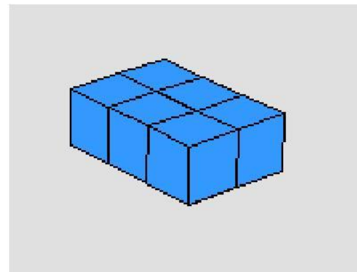
A rectangular aquarium will hold 24 cubic feet of water when filled to the top. If it is 4 ft. long and 3 ft. tall, how wide is the tank?	What is the volume of a rectangular prism with a height of 2 ft. and a base area of 6 ft^2?
Cube-shaped boxes of candy are shipped in larger boxes. The larger boxes are six feet long, one foot wide, and two feet high. How many one cubic foot boxes of candy will the large box hold?	A rectangular juice box contains 24 milliliters of apple juice. The box is 2 cm. high and 3 cm wide. What is the length of the juice box? (1 milliliter = 1 cubic centimeter)
A wading pool will be 2 m long and 5 m wide. If it will hold 30 cubic meters of water, how deep should this wading pool be?	A moving company is trying to store 1 cubic meter boxes in a storage room with a length of 5 m, width of 3 m and height of 2 m. How many boxes can fit in this space?
What is the volume of a cube with an edge that measures 3 cm?	A toy company is planning to market a set of wooden alphabet blocks. Each block is a cube with 1-cm edges. How many blocks will fit into a container that is 3 cm long, 2 cm wide, and 3 cm tall?

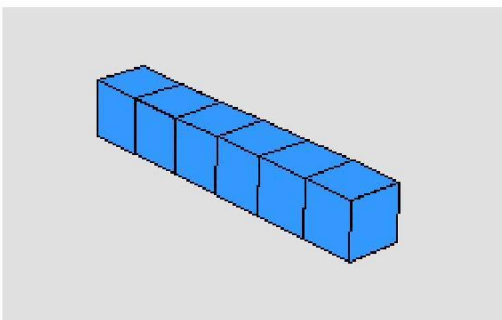
CARD SET B – 3-Dimensional Figures

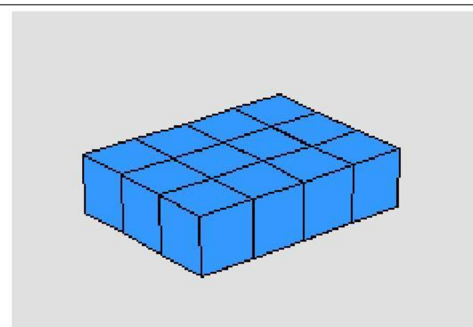
	
	
	
	

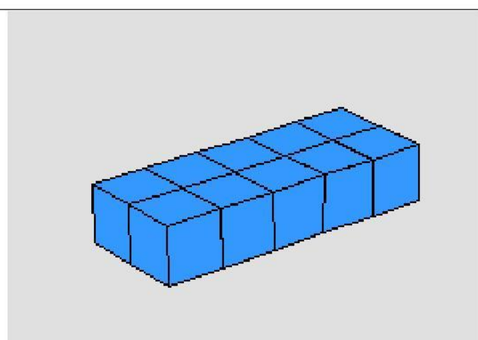
CARD SET C – Base Area

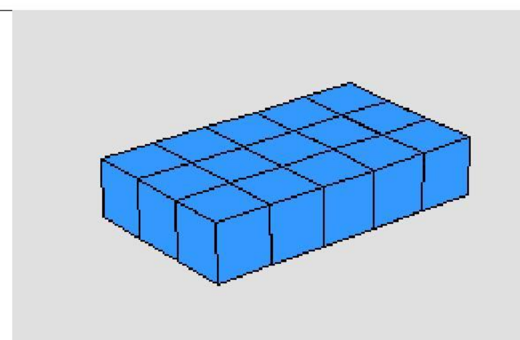


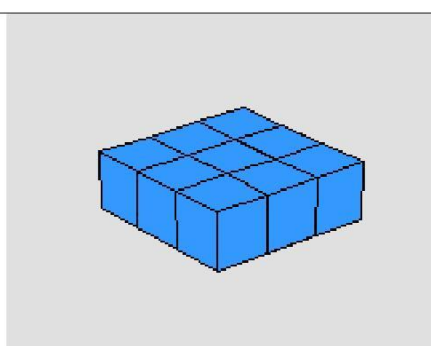


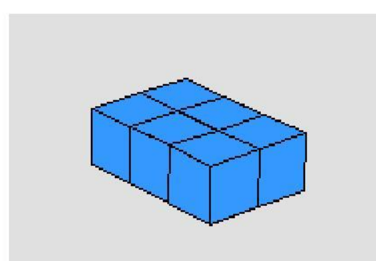












CARD SET D – V = L x W x H formula cards

24 = ___ x 3 x ___	**V = 5 x ___ x 3**
30 = __ x 3 x ___	**V = 4 x 2 x ___**
18 = __ x 2 x ___	**V = ___ x 2 x 2**
12 = 6 x ___ x ___	**V = 3 x 3 x ___**

Graphic Organizer to Record Ideas

Name:________________________________Date:__________________________

1. What did you notice?		
2. What do you wonder?	3. Main Question:	
4. Make an estimate.	5. Write an estimate that's too low.	6. Write an estimate that's too high.
7. Show your work.		
8. What is your conclusion?		