## North Carolina Department of Public Instruction

## INSTRUCTIONAL SUPPORT TOOLS

FOR ACHIEVING NEW STANDARDS

This document is designed to help North Carolina educators teach the Common Core (Standard Course of Study). NCDPI staff are continually updating and improving these tools to better serve teachers.

## Geometry • Unpacked Content

For the new Common Core standards that will be effective in all North Carolina schools in 2012-13.

## What is the purpose of this document?

To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do.

## What is in the document?

Descriptions of what each standard means a student will know, understand and be able to do. The "unpacking" of the standards done in this document is an effort to answer a simple question "What does this standard mean that a student must know and be able to do?" and to ensure the description is helpful, specific and comprehensive for educators.

## How do I send Feedback?

We intend the explanations and examples in this document to be helpful and specific. That said, we believe that as this document is used, teachers and educators will find ways in which the unpacking can be improved and made ever more useful. Please send feedback to us at feedback@dpi.state.nc.us and we will use your input to refine our unpacking of the standards. Thank You!

## Congruence

## Common Core Cluster

## Experiment with transformations in the plane.

## Common Core Standard

G.CO. 1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
G.CO. 2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO. 3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

## Unpacking

What does this standard mean that a student will know and be able to do?
G.CO. 1 Understand and use definitions of angles, circles, perpendicular lines, parallel lines, and line segments based on the undefined term of a point, a line, the distance along a line, and the length of an arc.
G.CO. 2 Use various technologies such as transparencies, geometry software, interactive whiteboards, and digital visual presenters to represent and compare rigid and size transformations of figures in a coordinate plane. Comparing transformations that preserve distance and angle to those that do not.
G.CO. 2 Describe and compare function transformations on a set of points as inputs to produce another set of points as outputs, to include translations and horizontal and vertical stretching.
G.CO. 3 Describe the rotations and reflections of a rectangle, parallelogram, trapezoid, or regular polygon that maps each figure onto itself.
G.CO. 4 Develop definitions of rotations, reflections, and translations in terms of angles, circles,
perpendicular lines, parallel lines, and line segments.
G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using,
e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## Instructional Expectations

The expectation for both pathways, in Geometry and CCSS Mathematics I, is to build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.

## Congruence

## Common Core Cluster

## Understand congruence in terms of rigid motions.

## Common Core Standard

G.CO. 6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
G.CO. 7 Use the definition of congruence in terms of rigid motions
to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
G.CO. 8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

## Unpacking

What does this standard mean that a student will know and be able to do?
G.CO.6 Use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane.
G.CO.6 Knowing that rigid transformations preserve size and shape or distance and angle, use this fact to connect the idea of congruency and develop the definition of congruent.
G.CO. 7 Use the definition of congruence, based on rigid motion, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.
G.CO. 8 Use the definition of congruence, based on rigid motion, to develop and explain the triangle congruence criteria; ASA, SSS, and SAS.

## Instructional Expectations

The expectation for both pathways, in Geometry and CCSS Mathematics I, is to understand that rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

## Congruence

## Common Core Cluster

## Prove geometric theorems.

Using logic and deductive reasoning, algebraic and geometric properties, definitions, and proven theorems to draw conclusions. (Encourage multiple ways of writing proofs, to include paragraph, flow charts, and two-column format)

## Common Core Standard

G.C0. 9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

## G.CO. 10 Prove theorems about

 triangles. Theorems include: measures of interior angles of a triangle sum to $180^{\circ}$; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
## Unpacking

What does this standard mean that a student will know and be able to do?
G.CO. 9 Prove theorems pertaining to lines and angles.

- Prove vertical angles are congruent.
- Prove when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angels are congruent.
- Prove points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
G.CO.10 Prove theorems pertaining to triangles.
- Prove the measures of interior angles of a triangle have a sum of $180^{\circ}$.
- Prove base angles of isosceles triangles are congruent.
- Prove the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length.
- Prove the medians of a triangle meet at a point.


## G.CO. 11 Prove theorems about parallelograms. Theorems include: <br> angles are congruent, the diagonals of <br> a parallelogram bisect each other, and <br> conversely, rectangles are <br> parallelograms with congruent <br> diagonals.

G.CO. 11 Prove theorems pertaining to parallelograms.

- Prove opposite sides are congruent.
- Prove opposite angles are congruent.
- Prove the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

In the traditional pathway, the expectation in Geometry is to encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of G.CO. 10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C. 3 in Unit 5.

In the international pathway, the expectation in CCSS Mathematics II is to encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of G.CO. 10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C. 3 in Unit 6.

## Congruence

## Common Core Cluster

## Make geometric constructions.

Create formal geometric constructions using a compass and straightedge, string, reflective devices, paper folding, and dynamic geometric software.

## Common Core Standard

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
G.CO. 13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

## Unpacking

What does this standard mean that a student will know and be able to do?
G.CO.12 Copy a segment
G.CO. 12 Copy an angle.
G.CO. 12 Bisect a segment
G.CO. 12 Bisect an angle
G.CO. 12 Construct perpendicular lines, including the perpendicular bisector of a line segment.
G.CO. 12 Construct a line parallel to a given line through a point not on the line.
G.CO. 13 Construct an equilateral triangle so that each vertex of the equilateral triangle is on the circle.
G.CO. 13 Construct a square so that each vertex of the square is on the circle.
G.CO.13 Construct a regular hexagon so that each vertex of the regular hexagon is on the circle.

## Instructional Expectations

In both pathways, for Geometry and CCSS Mathematics I, the expectation is to build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous
standards and can be introduced in conjunction with them.

## Similarity, Right Triangles, and Trigonometry

## Common Core Cluster

Understand similarity in terms of similarity transformations.

## Common Core Standard

G.SRT. 1 Verify experimentally the properties of dilations given by a center and a scale factor.
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is

## Unpacking

What does this standard mean that a student will know and be able to do?
G.SRT.1a Given a center and a scale factor, verify experimentally, that when dilating a figure in a coordinate plane, a segment of the pre-image that does not pass through the center of the dilation, is parallel to it's image when the dilation is preformed. However, a segment that passes through the center remains unchanged.
G.SRT.1b Given a center and a scale factor, verify experimentally, that when performing dilations of a line segment, the pre-image, the segment which becomes the image is longer or shorter based on the ratio given by the scale factor.

| longer or shorter in the ratio given <br> by the scale factor. |  |
| :--- | :--- |
| G.SRT. 2 Given two figures, use the <br> definition of similarity in terms of <br> similarity transformations to decide if <br> they are similar; explain using <br> similarity transformations the <br> meaning of similarity for triangles as <br> the equality of all corresponding pairs <br> of angles and the proportionality of all <br> corresponding pairs of sides. | G.SRT. 2 Use the idea of dilation transformations to develop the definition of similarity. <br> of corresponding angles and the proportionality of corresponding sides. |
| G.SRT. 3 Use the properties of their similarity based on the equality <br> similarity transformations to establish <br> the AA criterion for two triangles to <br> be similar. | G.SRT. 3 Use the properties of similarity transformations to develop the criteria for proving similar triangles; AA. |

## Similarity, Right Triangles, and Trigonometry

## Common Core Cluster

## Prove theorems involving similarity.

| Common Core Standard | Unpacking <br> What does this standard mean that a student will know and be able to do? |
| :---: | :---: |
| G.SRT. 4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. | G.SRT. 4 Use AA, SAS, SSS similarity theorems to prove triangles are similar. <br> G.SRT. 4 Use triangle similarity to prove other theorems about triangles <br> - Prove a line parallel to one side of a triangle divides the other two proportionally, and it's converse <br> - Prove the Pythagorean Theorem using triangle similarity. |
| G.SRT. 5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | G.SRT. 5 Using similarity theorems, prove that two triangles are congruent. <br> G.SRT. 5 Prove geometric figures, other than triangles, are similar and/or congruent. |

## Similarity, Right Triangles, and Trigonometry

## Common Core Cluster

## Define trigonometric ratios and solve problems involving right triangles.

| Common Core Standard | Unpacking <br> What does this standard mean that a student will know and be able to do? |
| :--- | :--- |
| G.SRT.6 Understand that by <br> similarity, side ratios in right triangles <br> are properties of the angles in the <br> triangle, leading to definitions of <br> trigonometric ratios for acute angles. | G.SRT.6 Using a corresponding angle of similar right triangles, show that the relationships of the side ratios are <br> the same, which leads to the definition of trigonometric ratios for acute angles. |
| G.SRT.7 Explain and use the <br> relationship between the sine and <br> cosine of complementary angles. | G.SRT.7 Explore the sine of an acute angle and the cosine of its complement and determine their relationship. |
| G.SRT.8 Use trigonometric ratios <br> and the Pythagorean Theorem to <br> solve right triangles in applied <br> problems.* | G.SRT.8 Apply both trigonometric ratios and Pythagorean Theorem to solve application problems involving right <br> triangles. |

## Similarity, Right Triangles, and Trigonometry

## Common Core Cluster

## Apply trigonometry to general triangles.

| Common Core Standard | Unpacking <br> What does this standard mean that a student will know and be able to do? |
| :--- | :--- |
| G.SRT.9 (+) Derive the formula <br> A= $=1 / 2 ~ a b$ sin (C) for the area of a <br> triangle by drawing an auxiliary line <br> from a vertex perpendicular to the <br> opposite side. | G.SRT.9 For a triangle that is not a right triangle, draw an auxiliary line from a vertex, perpendicular to the <br> opposite side and derive the formula, A= $1 / 2 \mathrm{ab}$ sin (C), for the area of a triangle, using the fact that the height of the <br> triangle is, h=a sin(C). |
| G.SRT.10 (+) Prove the Laws of <br> Sines and Cosines and use them to <br> solve problems. | G.SRT.10 Using trigonometry and the relationship among sides and angles of any triangle, such as $\sin (\mathrm{C})=(\mathrm{h} / \mathrm{a})$, <br> prove the Law of Sines. |
|  | G.SRT.10 Using trigonometry and the relationship among sides and angles of any triangle and the Pythagorean <br> Theorem to prove the Law of Cosines. |
| G.SRT.10 Use the Laws of Sines to solve problems. |  |
| G.SRT.10 Use the Laws of Cosines to solve problems. |  |


| G.SRT. 11 (+) Understand and apply <br> the Law of Sines and the Law of | G.SRT. 11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measures in right <br> triangles. <br> Cosines to find unknown <br> measurements in right and non-right <br> triangles (e.g., surveying problems, <br> resultant forces). |
| :--- | :--- |
| G.SRT. 11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measures in non-right <br> triangles. |  |

definitions of sine and cosine must be extended to obtuse angles.

## Circles

## Common Core Cluster

## Understand and apply theorems about circles.

## Common Core Standard

## G.C. 1 Prove that all circles are similar.

## G.C. 2 Identify and describe

 relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.G.C. 3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

## Unpacking

What does this standard mean that a student will know and be able to do?
G.C. 1 Using the fact that the ratio of diameter to circumference is the same for circles, prove that all circles are similar.
G.C. 2 Using definitions, properties, and theorems, identify and describe relationships among inscribed angles, radii, and chords. Include central, inscribed, and circumscribed angles.
G.C. 2 Understand that inscribed angles on a diameter are right angles.
G.C. 2 Understand that the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
G.C. 3 Construct inscribed circles of a triangle.
G.C. 3 Construct circumscribed circles of a triangle.
G.C. 3 Using definitions, properties, and theorems, prove properties of angles for a quadrilateral inscribed in a circle.
G.C. 4 Construct a tangent line from a point outside a given circle to the circle.

## Common Core Cluster

## Find arc lengths and areas of sectors of circles.

## Common Core Standard

## G.C. 5 Derive using similarity the

fact that the length of the arc
intercepted by an angle is
proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

## Unpacking

What does this standard mean that a student will know and be able to do?
G.C. 5 Use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius, identifying the constant of proportionality as the radian measure of the angle.
G.C. 5 Find the arc length of a circle.
G.C. 5 Using similarity, derive the formula for the area of a sector.
G.C. 5 Find the area of a sector in a circle.

## Instructional Expectations

In both pathways, Geometry and CCSS Mathematics II, the expectation is to emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.

## Expressing Geometric Properties with Equations

## Common Core Cluster

## Translate between the geometric description and the equation for a conic section.

## Common Core Standard

## G.GPE. 1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G.GPE. 2 Derive the equation of a parabola given a focus and a directrix.
G.GPE. 3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

## Unpacking

What does this standard mean that a student will know and be able to do?
G.GPE. 1 Use the Pythagorean Theorem to derive the equation of a circle, given the center and the radius.
G.GPE. 1 Given an equation of a circle, complete the square to find the center and radius of a circle.
G.GPE. 2 Given a focus and directrix, derive the equation of a parabola.
G.GPE. 2 Given a parabola, identify the vertex, focus, directrix, and axis of symmetry, noting that every point on the parabola is the same distance from the focus and the directrix.
G.GPE. 3 Given the foci, derive the equation of an ellipse, noting that the sum of the distances from the foci to any fixed point on the ellipse is constant, identifying the major and minor axis.
G.GPE. 3 Given the foci, derive the equation of a hyperbola, noting that the absolute value of the differences of the distances form the foci to a point on the hyperbola is constant, and identifying the vertices, center, transverse axis, conjugate axis, and asymptotes.

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Instructional Expectations
In the traditional pathway, in Geometry, G. GPE. 2 indicates the directrix should be parallel to a coordinate axis.
In the international pathway, in CCSS Mathematics III, the expectation is to connect the equations of circles and parabolas to prior work with quadratic equations. The directrix should be parallel to a coordinate axis.
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## Expressing Geometric Properties with Equations

## G.GPE

## Common Core Cluster

Use coordinates to prove simple geometric theorems algebraically.

## Common Core Standard

G.GPE. 4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle
centered at the origin and containing the point $(0,2)$.

## Unpacking

What does this standard mean that a student will know and be able to do?
G.GPE. 4 Use coordinate geometry to prove geometric theorems algebraically; such as prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{ } 3)$ lies on the circle centered at the origin and containing the point $(0,2)$.

| G.GPE. 5 Prove the slope criteria for <br> parallel and perpendicular lines and <br> use them to solve geometric problems <br> (e.g., find the equation of a line <br> parallel or perpendicular to a given <br> line that passes through a given <br> point). | G.GPE. 5 Using slope, prove lines are parallel or perpendicular <br> G.GPE. 5 Find equations of lines based on certain slope criteria such as; finding the equation of a line parallel or <br> perpendicular to a given line that passes through a given point. |
| :--- | :--- |
| G.GPE.6 Find the point on a directed <br> line segment between two given <br> points that partitions the segment in a <br> given ratio. | G.GPE. 6 Given two points, find the point on the line segment between the two points that divides the segment into <br> a given ratio. |
| G.GPE.7 Use coordinates to compute <br> perimeters of polygons and areas of <br> triangles and rectangles, e.g., using <br> the distance formula.. | G.GPE. 7 Use coordinate geometry and the distance formula to find the perimeters of polygons and the areas of <br> triangles and rectangles. |
| Instructional Expectations |  |
| In both pathways, in Geometry and CCSS Mathematics II, connect G.GPE. 1 to G.GPE.4 and reasoning with triangles is limited to right triangles; e.g., <br> derive the equation for a line through two points using similar right triangles. Relate work on parallel lines in G.GPE. to work on A.REI.5 in High School <br> Algebra I involving systems of equations having no solution or infinitely many solutions. G.GPE. 7 provides practice with the distance formula and its <br> connection with the Pythagorean theorem. |  | connection with the Pythagorean theorem.

In both pathways, G.GPE. 4 in Geometry and CCSS Mathematics III, indicates including proofs involving circles.

## Geometric Measurement and Dimension

## Common Core Cluster

## Explain volume formulas and use them to solve problems.

## Common Core Standard

G.GMD. 1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
G.GMD. 2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
G.GMD. 3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*

## Unpacking

What does this standard mean that a student will know and be able to do?
G.GMD. 1 Explain the formulas for the circumference of a circle and the area of a circle by determining the meaning of each term or factor.
G.GMD. 1 Explain the formulas for the volume of a cylinder, pyramid and cone by determining the meaning of each term or factor.
G.GMD. 2 Using Cavalieri's Principle, provide informal arguments to develop the formulas for the volume of spheres and other solid figures.
G.GMD. 3 Solve problems using volume formulas for cylinders, pyramids, cones, and spheres.

## Instructional Expectations

In both pathways, the expectation for Geometry and CCSS Mathematics II is the understanding that informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k , its area is k 2 times the area of the first. Similarly, volumes of solid figures scale by k3 under a similarity transformation with scale factor k .

## Geometric Measurement and Dimension

## G.GMD

## Common Core Cluster

## Visualize relationships between two-dimensional and three-dimensional objects.

## Common Core Standard

## Unpacking

What does this standard mean that a student will know and be able to do?
G.GMD. 4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of twodimensional objects.
G.GMD. 4 Given a three- dimensional object, identify the shape made when the object is cut into cross-sections.
G.GMD. 4 When rotating a two- dimensional figure, such as a square, know the three-dimensional figure that is generated, such as a cylinder. Understand that a cross section of a solid is an intersection of a plane (twodimensional) and a solid (three-dimensional).

## Modeling with Geometry

## Common Core Cluster

## Apply geometric concepts in modeling situations.

## Common Core Standard

## Unpacking

What does this standard mean that a student will know and be able to do?
G.MG. 1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a
cylinder).*
G.MG. 2 Apply concepts of density based on area and volume in
modeling situations (e.g., persons per square mile, BTUs per cubic foot).*
G.MG. 3 Apply geometric methods to solve design problems (e.g. designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

## Instructional Expectations

