North Carolina Department of Public Instruction
INSTRUCTIONAL SUPPORT TOOLS
FOR ACHIEVING NEW STANDARDS

This document is designed to help North Carolina educators teach the Common Core (Standard Course of Study). NCDPI staff are continually updating and improving these tools to better serve teachers.

Algebra ● Unpacked Content

For the new Common Core standards that will be effective in all North Carolina schools in the 2012-13 school year.

What is the purpose of this document?
To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do.

What is in the document?
Descriptions of what each standard means a student will know, understand and be able to do. The "unpacking" of the standards done in this document is an effort to answer a simple question "What does this standard mean that a student must know and be able to do?" and to ensure the description is helpful, specific and comprehensive for educators.

How do I send Feedback?
We intend the explanations and examples in this document to be helpful and specific. That said, we believe that as this document is used, teachers and educators will find ways in which the unpacking can be improved and made ever more useful. Please send feedback to us at feedback@dpi.state.nc.us and we will use your input to refine our unpacking of the standards. Thank You!

Seeing Structure in Expressions A.SSE

Common Core Cluster

Interpret the structure of expressions

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
A.SSE.1 Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.	**A.SSE.1a** Identify the different parts of the expression and explain their meaning within the context of a problem. **A.SSE.1b** Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.
A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.*	**A.SSE.2** Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms. • Use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely. • Simplify expressions including combining like terms, using the distributive property and other operations with polynomials.

Instructional Expectations
In the **traditional pathway**, linear, quadratic, and exponential expressions are the focus in Algebra I, and integer exponents are extended to rational exponents (only those with square or cubed roots). In Algebra II, the expectation is to extend to polynomial and rational expressions. In the **international pathway**, CCSS Mathematics I focuses on linear expressions and exponential expressions with integer exponents. CCSS Mathematics II extends exponential expressions from the first course and includes quadratic expressions. Course II also extends integer exponents to rational exponents, focusing on those with square or cubed roots. In CCSS Mathematics III the expectation is polynomial and rational expressions.

Seeing Structure in Expressions A-SSE

Common Core Cluster

Write expressions in equivalent forms to solve problems

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.	**A.SSE.3a** Write expressions in equivalent forms by factoring to find the zeros of a quadratic function and explain the meaning of the zeros. • Given a quadratic function explain the meaning of the zeros of the function. That is if $f(x) = (x – c) (x – a)$ then $f(a) = 0$ and $f(c) = 0$. • Given a quadratic expression, explain the meaning of the zeros graphically. That is for an expression $(x – a) (x – c)$, *a* and *c* correspond to the *x*-intercepts (if *a* and *c* are real). **A.SSE.3b** Write expressions in equivalent forms by completing the square to convey the vertex form, to find the maximum or minimum value of a quadratic function, and to explain the meaning of the vertex. **A.SSE.3c** Use properties of exponents (such as power of a power, product of powers, power of a product, and rational exponents, etc.) to write an equivalent form of an exponential function to reveal and explain specific information about its approximate rate of growth or decay.

A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.	**A.SSE.4** Develop the formula for the sum of a finite geometric series when the ratio is not 1. **A.SSE.4** Use the formula to solve real world problems such as calculating the height of a tree after *n* years given the initial height of the tree and the rate the tree grows each year. Calculate mortgage payments.

Instructional Expectations

It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal. Also, consider extending finite geometric series to infinite geometric series for enrichment.

In the **traditional pathway**, quadratic, and exponential expressions are the focus in Algebra I. The expectation in Algebra II is to extend to polynomial and rational expressions.

In the **international pathway**, CCSS Mathematics II focuses on quadratic and exponential expressions, while in CCSS Mathematics III the expectation is polynomial and rational expressions.

Arithmetic With Polynomials and Rational Expressions — A.APR

Common Core Cluster

Perform arithmetic operations on polynomials

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	**A.APR.1** Understand the definition of a polynomial. **A.APR.1** Understand the concepts of combining like terms and closure. **A.APR.1** Add, subtract, and multiply polynomials and understand how closure applies under these operations.

Instructional Expectations

Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.

In the **traditional pathway,** linear and quadratic polynomial expressions are the expectation in Algebra I and beyond quadratic polynomial expressions is the expectation for Algebra II.

In the **international pathway,** CCSS Mathematics II places emphasis on polynomials that simplify to quadratics and CCSS Mathematics III extends beyond quadratics.

Arithmetic With Polynomials and Rational Expressions — A.APR

Common Core Cluster

Understand the relationship between zeros and factors of polynomials

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
A.APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.	**A.APR.2** Understand and apply the Remainder Theorem. **A.APR.2** Understand how this standard relates to A.SSE.3a. **A.APR.2** Understand that a is a root of a polynomial function if and only if x-a is a factor of the function.
A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	**A.APR.3** Find the zeros of a polynomial when the polynomial is factored. **A.APR.3** Use the zeros of a function to sketch a graph of the function.

Arithmetic With Polynomials and Rational Expressions	**A.APR**
Common Core Cluster	
Use polynomial identities to solve problems	
Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
A.APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	**A.APR.4** Understand that polynomial identities include but are not limited to the product of the sum and difference of two terms, the difference of two squares, the sum and difference of two cubes, the square of a binomial, etc . **A.APR.4** Prove polynomial identities by showing steps and providing reasons. **A.APR.4** Illustrate how polynomial identities are used to determine numerical relationships such as $25^2 = (20+5)^2 = 20^2 + 2 \bullet 20 \bullet 5 + 5^2$
A.APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle.[1] [1]The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.	**A.APR.5** For small values of *n*, use Pascal's Triangle to determine the coefficients of the binomial expansion. **A.APR.5** Use the Binomial Theorem to find the nth term in the expansion of a binomial to a positive power.
Instructional Expectations	

This cluster has many possibilities for optional enrichment, such as relating the example in A.APR.4 to the solution of the system $u^2+v^2=1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x+y)^{n+1} = (x+y)(x+y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.

Arithmetic With Polynomials and Rational Expressions — A.APR

Common Core Cluster

Rewrite rational expressions

Common Core Standard	Unpacking What does this standard mean that a student will know and be able to do?
A.APR.6 Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x)*, where *a(x)*, *b(x)*, *q(x)*, and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system.	**A.APR.6** Rewrite rational expressions, $\frac{a(x)}{b(x)}$, in the form $q(x)+\frac{r(x)}{b(x)}$ by using factoring, long division, or synthetic division. Use a computer algebra system for complicated examples to assist with building a broader conceptual understanding.
A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	**A.APR.7** Simplify rational expressions by adding, subtracting, multiplying, or dividing. **A.APR.7** Understand that rational expressions are closed under addition, subtraction, multiplication, and division (by a nonzero expression).

Instructional Expectations

The limitations on rational functions apply to the rational expressions in A.APR.6. Standard A.APR.7 requires the general division algorithm for polynomials.

Creating Equations — A.CED

Common Core Cluster

Create equations that describe numbers or relationships

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	**A.CED.1** Create linear, quadratic, rational and exponential equations and inequalities in one variable and use them in a contextual situation to solve problems.
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	**A.CED.2** Create equations in two or more variables to represent relationships between quantities. **A.CED.2** Graph equations in two variables on a coordinate plane and label the axes and scales.
A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. For example, represent inequalities describing	**A.CED.3** Write and use a system of equations and/or inequalities to solve a real world problem. Recognize that the equations and inequalities represent the constraints of the problem. Use the Objective Equation and the Corner Principle to determine the solution to the problem. (Linear Programming)

nutritional and cost constraints on combinations of different foods.	
A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.	**A.CED.4** Solve multi-variable formulas or literal equations, for a specific variable.

Reasoning with Equations and Inequalities — A.CED

Common Core Cluster

Understand solving equations as a process of reasoning and explain the reasoning

Common Core Standard	Unpacking What does this standard mean that a student will know and be able to do?
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**A.REI.1** Assuming an equation has a solution, construct a convincing argument that justifies each step in the solution process. Justifications may include the associative, commutative, and division properties, combining like terms, multiplication by 1, etc.
A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**A.REI.1** Solve simple rational and radical equations in one variable and provide examples of how extraneous solutions arise.

Instructional Expectations

In the **traditional pathway, Algebra I,** students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Algebra II. In **Algebra II**, extend to simple rational and radical equations.

In the **international pathway**, **CCSS Mathematics I**, students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms and extend their work to simple rational and radical equations in **CCSS Mathematics III**.

Reasoning with Equations and Inequalities	**A.CED**
Common Core Cluster	
Solve equations and inequalities in one variable	
Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**A.REI.3** Solve linear equations in one variable, including coefficients represented by letters. **A.REI.3** Solve linear inequalities in one variable, including coefficients represented by letters.
A.REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	**A.REI.4a** Transform a quadratic equation written in standard form to an equation in vertex form $(x - p)^2 = q$ by completing the square. **A.REI.4a** Derive the quadratic formula by completing the square on the standard form of a quadratic equation.
A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.	**A.REI.4b** Solve quadratic equations in one variable by simple inspection, taking the square root, factoring, and completing the square. **A.REI.4b** Understand why taking the square root of both sides of an equation yields two solutions. **A.REI.4b** Use the quadratic formula to solve any quadratic equation, recognizing the formula produces all complex solutions. Write the solutions in the form $a \pm bi$, where *a* and *b* are real numbers. **A.REI.4b** Explain how complex solutions affect the graph of a quadratic equation.
Instructional Expectations	

In the **traditional pathway**, **Algebra I** students should extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as 5x=125 or 2x=1/16. Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.

In the **international pathway**, **CCSS Mathematics I** students extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as 5x=125 or 2x=1/16. In **CCSS Mathematics II**, students extend to solving any quadratic equation with real coefficients, including those with complex solutions.

Reasoning with Equations and Inequalities	**A.CED**
Common Core Cluster	
Solve systems of equations	
Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	**A.REI.5** Solve systems of equations using the elimination method (sometimes called linear combinations). **A.REI.5** Solve a system of equations by substitution (solving for one variable in the first equation and substitution it into the second equation).
A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	**A.REI.6** Solve systems of equations using graphs.
A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.	**A.REI.7** Solve a system containing a linear equation and a quadratic equation in two variables (conic sections possible) graphically and symbolically.
A.REI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.	**A.REI.8** Write a system of linear equations as a single matrix equation.

A.REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	**A.REI.9** Find the inverse of the coefficient matrix in the equation, if it exits. Use the inverse of the coefficient matrix to solve the system. Use technology for matrices with dimensions 3 by 3 or greater. • Find the dimension of matrices. • Understand when matrices can be multiplied. • Understand that matrix multiplication is not commutative. • Understand the concept of an identity matrix. • Understand why multiplication by the inverse of the coefficient matrix yields a solution to the system (if it exists).

Instructional Expectations

Standards with matrices are reserved for a fourth course.

In the **traditional pathway**, **Algebra I** students should build on experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to GPE.5 when it is taught in Geometry, which requires students to prove the slope criteria for parallel lines. Also include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2+y^2=1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2+4^2=5^2$.

In the **international pathway**, **CCSS Mathematics I** students build on experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to GPE.5 when it is taught in Geometry, which requires students to prove the slope criteria for parallel lines. In **CCSS Mathematics II**, include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.

Reasoning with Equations and Inequalities — A.CED

Common Core Cluster

Represent and solve equations and inequalities graphically

Common Core Standard	Unpacking What does this standard mean that a student will know and be able to do?
A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	**A.REI.10** Understand that all solutions to an equation in two variables are contained on the graph of that equation.
A.REI.11 Explain why the x-coordinates of the points where the graphs of the *equations* $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	**A.REI.11** Explain why the intersection of $y = f(x)$ and $y = g(x)$ is the solution of $f(x) = g(x)$ for any combination of linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Find the solution(s) by: • Using technology to graph the equations and determine their point of intersection, • Using tables of values, or • Using successive approximations that become closer and closer to the actual value.

A.REI.12 Graph the solutions to a linear inequality in two variables as a half- plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane, excluding the boundary for non-inclusive inequalities. **A.REI.12** Graph the solution set to a system of linear inequalities in two variables as the intersection of their corresponding half-planes.

Instructional Expectations

In the **traditional pathway**, for A.REI.10, **Algebra I** students should focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses. For A.REI.11, focus on cases where *f(x)* and *g(x)* are linear or exponential. **Algebra II** students should include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.

In the **international pathway**, **CCSS Mathematics I** students should focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses. For A.REI.11, focus on cases where *f(x)* and *g(x)* are linear or exponential. CCSS Mathematics III students should include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.